Original Article

A Theoretical Analysis of Dollar-Cost Averaging Under Different Index Price Trajectories

-- Mathematical Conditions, Price Trajectories, Volatility, Recovery, and Investor Behavior

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Abstract

Dollar-cost averaging (DCA) is the practice of investing a fixed dollar amount at regular intervals regardless of the current market price. This paper develops a simple theoretical framework for DCA under different index price trajectories. The analysis begins with a break-even condition: for a completed DCA program to be profitable at liquidation, the final asset price must exceed the investor's average cost per share. For equal-dollar contributions, this average cost is the harmonic mean of the purchase prices. The paper then examines linear and exponential growth, temporary declines followed by recovery, irregular market paths, and permanent impairment. A key clarification is made between profit as a percentage of cumulative contributions and an annualized money-weighted return. Under continuous equal-rate investing into an index that rises linearly from 100 to 110 over one year, ending wealth is approximately 1.0484 times total contributions, a 4.84% gain on contributed capital; this should not be called a 4.84% annualized return because the contributions were invested for different lengths of time. The framework further shows why DCA can benefit from temporary price weakness if recovery occurs, while permanent deterioration remains destructive. The practical implication is that long-term DCA is most coherent when applied to a broadly diversified asset with durable expected growth, low probability of permanent loss, and sufficient investor conviction to maintain the strategy through severe drawdowns.

Keywords: dollar-cost averaging; index investing; average cost; harmonic mean; volatility; drawdown; money-weighted return; investor behavior

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1. Introduction

Dollar-cost averaging is widely used in retirement plans and other periodic investment programs. Its defining feature is not an attempt to forecast short-term prices, but a rule: invest a fixed dollar amount on a predetermined schedule. Because a fixed contribution purchases more shares when prices are low and fewer shares when prices are high, DCA creates a distinctive relationship between the path of prices and the investor's accumulated cost.

This analysis begins with two observations. First, a DCA program is profitable at a chosen terminal date when the terminal price exceeds the accumulated average cost. Second, the psychological feasibility of continuing to invest depends heavily on confidence that the underlying asset can survive declines and resume long-term growth. To connect the mathematics with investment behavior, price histories are organized into five intuitive cases: an ideal low-buy/high-sell path, exponential growth, linear growth, irregular real-world behavior, and a best practical case involving a long-term-growing index with a low probability of permanent loss.

The framework combines intuitive price-path reasoning with formal mathematics. In particular, it distinguishes (i) gain relative to cumulative contributions, (ii) time-weighted asset return, and (iii) money-weighted investor return. This distinction matters because periodic contributions enter the market at different dates.

2. Mathematical framework

2.1 Equal-dollar periodic investment

Suppose an investor contributes the same amount C at n purchase dates. Let P_i denote the asset price at purchase i. The number of shares purchased at that date is $\frac{C}{P_i}$. Total shares accumulated are therefore:

$$Q = C \sum_{i=1}^{n} \frac{1}{P_i}$$

Total contributed capital is nC. The investor's average cost per share is:

$$P_{avg} = \frac{nC}{Q} = \frac{n}{\sum_{i=1}^{n} \frac{1}{P_i}}$$

Thus, for equal-dollar contributions, the average cost per share is the harmonic mean of the purchase prices. It is generally not the arithmetic mean.

2.2 Break-even and profit condition

Let P_f be the terminal price at which the accumulated position is valued or liquidated. Terminal wealth is QP_f, and profit is $QP_f - nC$. Since $nC = QP_{avg}$:

$$Profit = Q(P_f - P_{avg})$$

Therefore:

$$Profit > 0 \quad \Leftrightarrow \quad P_f > P_{avg}$$

This break-even relation is a terminal accounting identity, not a forecast that DCA must be profitable.

3. Linear growth: The 10% one-year example

Consider an index that begins at 100 and rises linearly to 110 over one year. Let t range from 0 to 1:

$$P(t) = 100(1 + 0.1t)$$

Assume contributions occur continuously at a constant rate and normalize total annual contributions to 1. The shares accumulated are:

$$Q = \int_0^1 \frac{dt}{100(1 + 0.1t)} = \frac{ln(1.1)}{10}$$

At the year-end price of 110, ending wealth is:

$$V = 110Q = \frac{110\ ln(1.1)}{10} \approx 1.04841$$

Hence the investor has approximately 4.84% more wealth than cumulative contributions. A rough midpoint estimate gives about 4.5%-5%; the continuous-contribution calculation gives 4.84% under these assumptions.

3.1 Why 4.84% is not an annualized return

Calling 4.84% an 'annual return' is potentially misleading. The first contribution is invested for almost the full year, while the last contribution is invested for almost no time. Dividing terminal profit by all contributions treats every dollar as though it had the same investment horizon.

A money-weighted return (IRR or XIRR, i.e. Internal Rate of Return or Extended Internal Rate of Return) accounts for the timing of each cash flow. If the asset follows a smooth return process consistent with a 10% annual return, the properly annualized investor return will be close to the asset's annual return; the lower 4.84% figure mainly reflects the fact that the average dollar was invested for only part of the year, as shown in Table 1.

Table 1: Index return vs DCA return

Measure	Approximate result	Interpretation
Index start-to-end return	10.0%	100 → 110
DCA gain / total contributions	4.84%	Ending wealth relative to all cash contributed
Annualized investor return	≈ underlying annual rate*	Must account for cash-flow timing

*Exact money-weighted return depends on the timing convention and discrete versus continuous contributions.

4. Linear versus exponential growth

A linear path adds a constant absolute amount over equal time intervals, whereas an exponential path compounds. For example, a 10% compounded process generates 100, 110, 121, 133.1, and so on, while a simple linear path might generate 100, 110, 120, 130. Over short intervals, exponential growth can be approximated by a linear function because $e^x = 1 + x$ for small x. Over long horizons, however, compounding creates increasingly large differences.

A broad equity index may therefore look approximately linear over a short interval, while long-run wealth accumulation is better understood through compounded growth. Actual index prices, however, are stochastic and are neither linear nor smoothly exponential.

5. Five price-trajectory cases for DCA

The five cases below separate mathematical attractiveness from psychological investability. The ideal mathematical path is not necessarily the easiest path to hold. A steadily rising market builds confidence but raises later purchase prices; a volatile market may create attractive purchase opportunities but tests investor conviction. The most useful long-term case therefore combines durable growth, recoverability after drawdowns, and a low probability of permanent loss (Figure 1).

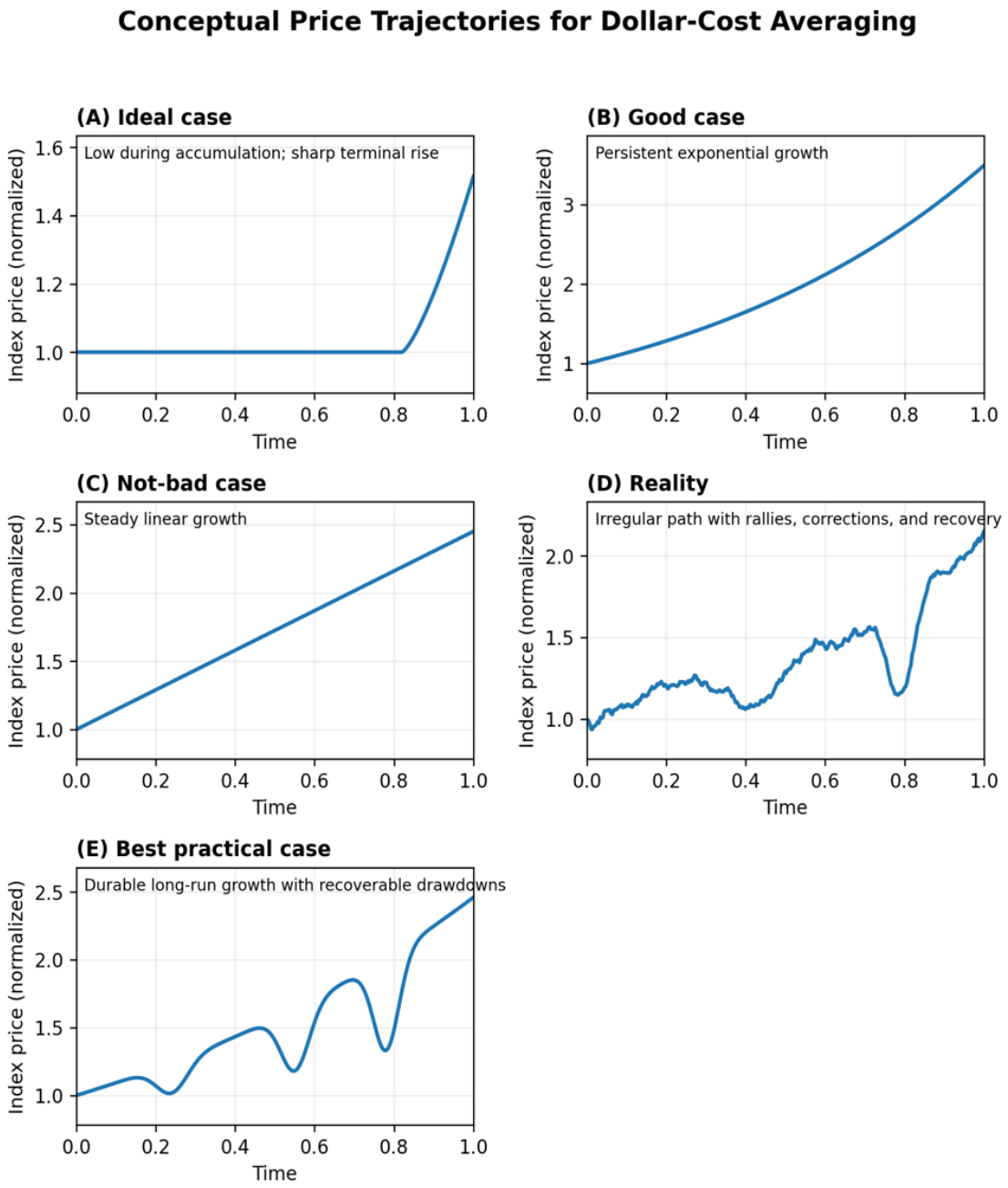


Figure 1. Five conceptual price trajectories for dollar-cost averaging. The curves illustrate qualitative investment cases rather than observed market data

5.1. Ideal case: buy very low, exit very high, as shown in Figure 1A

The mathematically attractive path is one in which most contributions occur at depressed prices and the terminal price is very high. Such a path maximizes shares accumulated per contributed dollar. Its weakness is behavioral and informational: a long period of low prices

can make it difficult to know whether the asset is temporarily undervalued or permanently impaired.

5.2. Good case: persistent exponential growth, as shown in Figure 1B

A steadily compounding asset reinforces investor confidence because new highs occur repeatedly. The disadvantage for DCA is that later contributions buy fewer shares. Nevertheless, a sufficiently high terminal price can dominate the higher purchase costs and produce strong long-run wealth accumulation.

5.3. Not-bad case: linear growth, as shown in Figure 1C

A steadily rising linear path has similar behavioral advantages but lacks the accelerating terminal value produced by compounding. Over a short horizon this can be a useful approximation; over long horizons it understates the economic significance of compound growth.

5.4. Reality: irregular and difficult to predict, as shown in Figure 1D

Actual markets combine rallies, corrections, crashes, recoveries, plateaus, and regime changes. The important question for DCA is therefore not whether the path is smooth, but whether the asset's long-run economic value and terminal price are sufficiently resilient relative to the investor's accumulated cost.

5.5. Best practical case: durable growth with low permanent-loss probability, as shown in Figure 1E

For a long-horizon investor, the most favorable practical target is a diversified asset with durable expected growth and a low probability of permanent destruction of capital. This condition is also behavioral: an investor who believes the asset can survive severe drawdowns is more likely to continue buying when DCA mathematically benefits most from lower prices.

6. Volatility, recovery, and the central asymmetry of DCA

DCA has an important asymmetry: temporary declines can be beneficial if they are followed by recovery, but permanent declines are harmful. The mechanism is mechanical.

When price falls, a fixed contribution purchases more shares. If the price later recovers, those low-cost shares contribute disproportionately to terminal wealth.

6.1. Simple recovery example

Table 2: recovery example

Stage	Price	Contribution	Shares purchased
Purchase 1	$100	$1,000	10
Purchase 2	$50	$1,000	20
Terminal valuation	$100	—	30 shares × $100 = $3,000

As shown in Table 2, total contributions are $2,000 and terminal wealth is $3,000, a 50% gain relative to contributed capital, even though the asset's start-to-end price return is 0%. This does not mean volatility automatically creates value. The result depends critically on the recovery to 100, as shown by the blue line in Figure 2.

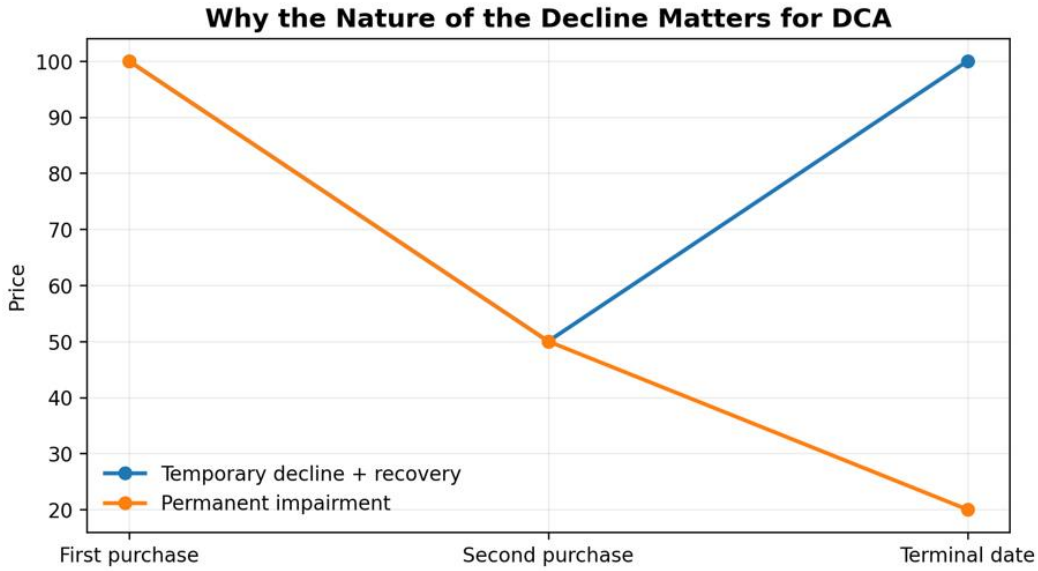


Figure 2. Temporary decline followed by recovery versus permanent impairment. Lower interim prices can improve DCA accumulation only when subsequent prices recover sufficiently. DCA can benefit from temporary price weakness, but it cannot rescue an asset from permanent impairment.

6.2. Permanent decline

If the same sequence were 100 → 50 → 20 with no recovery, the additional shares purchased at 50 would not convert the strategy into a success, as shown by the orange line in Figure 2. Repeated purchases can increase exposure to a deteriorating asset. Consequently, 'buying the dip' and DCA are only defensible when the underlying investment thesis remains valid.

7. Investor behavior and strategy persistence permanent decline

Investor confidence is a central part of the theory. A mathematically attractive strategy can fail in practice if the investor abandons it during a drawdown. DCA therefore has a behavioral dimension: a rules-based schedule may reduce the temptation to time each purchase, while diversification and confidence in the underlying asset may improve the probability that the investor continues the plan.

External research supports the relevance of this trade-off. Vanguard reports that, when an investor already possesses a lump sum, immediate lump-sum investment historically outperformed common cost-averaging approaches roughly two-thirds of the time, reflecting the opportunity cost of holding part of the capital in cash. At the same time, cost averaging may be attractive to highly loss-averse investors because it reduces initial market exposure and can make adherence easier (Finlay & Zorn, 2023).

This literature addresses a different question from paycheck-based periodic investing. When future savings are not yet available, investing each contribution as it arrives does not involve deliberately withholding an already-available lump sum. The distinction is essential when interpreting empirical comparisons between DCA and lump-sum investing.

8. Relationship to existing research

Academic research has often found that DCA sacrifices expected return relative to immediate lump-sum investment when all capital is available at the outset, because DCA delays exposure to assets with a positive expected risk premium. Knight and Mandell (1992) reported unfavorable results for DCA using analytical, historical, and Monte Carlo approaches. More recent formal work by Kirkby, Mitra, and Nguyen (2020) derives expected wealth and variance for DCA-type strategies and shows that DCA can reduce variance under suitable assumptions, but at the cost of lower expected returns.

Risk, however, is multidimensional. Trainor (2005) examined within-horizon exposure to loss and found that DCA can reduce the probability, magnitude, and duration of large interim losses relative to lump-sum investing. Thus, the choice between DCA and lump-sum investment cannot be summarized solely by average terminal return; it also depends on the investor's cash availability, loss tolerance, interim liquidity needs, and ability to maintain the strategy.

9. Refined propositions

Proposition 1 — Break-even condition. For equal-dollar DCA, terminal profit is positive if and only if the terminal price exceeds the harmonic-mean average cost of the purchased shares.

Proposition 2 — Contribution return is not annualized return. Profit divided by cumulative contributions is a useful accounting measure but is not a valid annualized return when contributions occur at different dates.

Proposition 3 — Path dependence. For a fixed terminal price, the sequence of intermediate prices affects the number of shares accumulated and therefore affects terminal DCA wealth.

Proposition 4 — Temporary weakness can help. Holding other factors constant, lower intermediate purchase prices can improve terminal DCA wealth when the asset subsequently recovers.
Proposition 5 — Permanent impairment dominates averaging. DCA cannot transform a permanently deteriorating asset into a sound investment merely by lowering average cost.
Proposition 6 — Asset selection and persistence are linked. A long-term DCA program is most robust when the underlying asset is sufficiently diversified and durable that the investor can rationally maintain contributions through major drawdowns.

10. Practical implications

The framework suggests that DCA should not be understood as a mechanism that guarantees profit or mechanically turns volatility into return. Its main mechanical benefit is the acquisition of more shares at lower prices. Whether this becomes economically valuable depends on subsequent prices.
For broad, diversified equity indexes, the rationale for long-term periodic investing is therefore stronger than for a single fragile company or a highly speculative asset. Diversification reduces idiosyncratic failure risk, while long investment horizons allow the investor to participate in economic growth. Even then, no index is guaranteed to rise, and historical resilience does not eliminate future risk.
The theory also discourages an overly simplistic preference for smooth price appreciation. A path with temporary declines may produce better DCA accumulation than a monotonic path with the same terminal price, provided recovery occurs. The key distinction is not 'volatility versus no volatility' but 'recoverable volatility versus permanent loss.

11. Practical implications

The analysis is intentionally stylized. It ignores taxes, transaction costs, dividends, inflation, bid-ask spreads, changing contribution amounts, interest earned on uninvested cash, sequence-dependent withdrawals, and stochastic changes in expected returns. The linear and exponential examples are deterministic illustrations rather than models of actual market dynamics.
Future work could test the propositions with historical total-return data for broad indexes, simulate stochastic price processes, compare monthly and biweekly contribution schedules, incorporate dividends and inflation, and measure outcomes using XIRR, terminal wealth, maximum drawdown, shortfall probability, and investor utility.

12. Conclusions

Dollar-cost averaging is best understood as a cash-flow rule whose outcome depends on both the terminal price and the path taken to reach it. For equal-dollar contributions, the relevant average purchase cost is the harmonic mean of purchase prices, and terminal profitability requires the final price to exceed that cost. In a one-year linear rise from 100 to

110 with continuous equal-rate contributions, terminal wealth is about 4.84% above cumulative contributions; this figure is not, however, the investor's annualized return.

The most important theoretical insight is the asymmetry between temporary and permanent declines. Temporary weakness followed by recovery allows fixed contributions to acquire more shares at favorable prices. Permanent impairment makes the same averaging mechanism destructive. Thus, the central requirement for a long-term DCA strategy is not the absence of volatility, but a credible basis for long-term growth and recovery. DCA benefits from recoverable weakness, but it cannot overcome permanent impairment.

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Data availability

The data that support the findings of this study are available from the author upon reasonable request.

Declaration of competing interest

The author declares that he has no competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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